Optimization Material Distribution methodology: Some electromagnetic examples

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***Abstract*—In this paper, a new approach towards Optimization Material Distribution (OMD) is proposed. The principle behind this approach is to use a surface genetic algorithm to determine material distribution within a fixed Finite Element Computation mesh. The method proves to be highly adaptive to various kinds of electromagnetic actuator optimization approaches. Several optimal electromagnetic examples are presented.**

***Index Terms*—Optimization, Design, OMD, Electromagnetic actuators**

I. Operating principle

Generally speaking, the design process adopted for technical objects such as actuators entails "human"-scale iterations. The inherent architecture of the object (i.e. how materials are laid out) does not result from any mathematical optimization due to the difficulty involved in establishing a formulation as part of a more classical approach. The optimization strategy quite often therefore gets detached from design. In reality, optimization only enters the design process later on as a means of quantifying, a posteriori, and then validating or not the pertinence of this structural choice. Within this classical approach (so-called "shape optimization", see Fig. 1), optimization variables consist of the geometrical parameters of the selected structure (e.g. airgap radius, yoke thickness) once its overall shape has been defined. This approach is thus intrinsically tied to the various prerequisites incorporated by the designer, which naturally introduces a certain number of deficiencies, namely:

- optimality of the selected structure is not guaranteed;
- the range of potential architectural solutions is narrow since the approach is being dictated by human-motivated activity;
- for new applications (especially in MEMS, wherein the designer does not necessarily have to meet prerequisites, it is difficult to derive well-adapted solutions.

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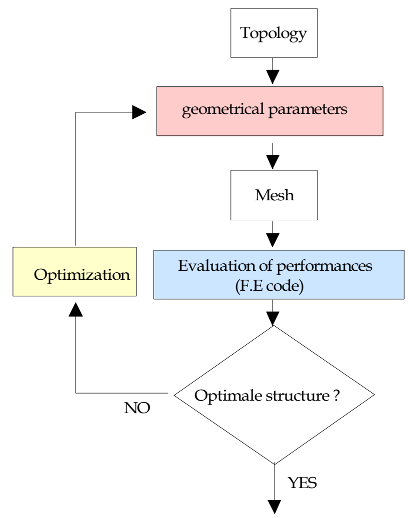


Fig. 1. Shape optimization flowchart, according to the classical approach

These few disadvantages may, in part, be overcome through use of an entirely quantitative design procedure, called "**Optimization Material Distribution**" (OMD). According to this approach, *design/optimization* processes overlap; topological constraints are scaled back to the bare minimum. Moreover, discontinuous material distribution becomes feasible, which is capable of leading to an optimal solution; such a methodological step would have been difficult to imagine when considering a strictly continuous distribution. The steps employed can indeed give rise to a *bona fide* generic Design Aid tool, providing the designer with a high-performance resource for extending the field of investigation and expanding the array of inventive capabilities.

Figure 2 presents a flowchart illustrating the OMD method. As opposed to classical strategy, the optimization variables here consist of the physical characteristics of each mesh link (e.g. resistivity, permeability, current density). The mesh thus corresponds to the discretization of both the finite element geometry and the material distribution. This mesh remains unchanged regardless of topology; the electrical and magnetic properties of each element undergo modification within the OMD optimization process. The number of variables is hence proportional to the product of the number of elements times the number of physical variables. The discretization of each physical magnitude may also be taken into consideration, in which case the focus lies in optimizing the distribution functions and not the actual distribution itself.

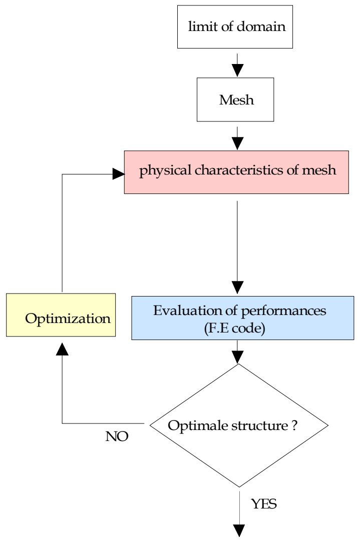


Fig. 2. OMD flowchart

Optimization constraints tend to be of the three following types:

- volume constraints;
- physical constraints correlated with the limit characteristics of the materials employed (e.g. saturation, temperature rise, demagnetization);
- so-called functional constraints, corresponding to spaces whose characteristics have been specifically set to perform a given function, e.g. rotation of the mobile part.

With respect to the optimization methodology, we employed a genetic algorithm approach [1], which seemed especially well-suited to this type of problem.

Since the mid-1990's, several laboratories throughout the world have been developing this methodology. The efforts undertaken by N. Dyck [2], in designing an elementary magnetic bearing with induced current by means of OMD, merit special mention. The objective behind the OMD method applied was to optimize material distribution (both iron and copper) in order to generate a supporting force of a given minimum value and to minimize the bearing mass and Joule losses. In [3], S. Wang applied the OMD method in the case of an electro-thermomechanical microactuator. In this instance, the objective concerned optimizing the distribution of a thermoelastic material, featuring given characteristics and contained within a fixed volume, to allow maximizing horizontal displacement for an imposed spring stiffness. Next, in [4], S. Dufour optimized distribution of the ferromagnetic material from a rotor on a variable reluctance synchronous machine so as to maximize average torque.

In the present article, we applied this method to various simple electromagnetic problems for the purpose of testing OMD relevance. In the following sections, we will describe several of the examples studied.

II. First example: Ferromagnetic electromagnet

One procedure for producing Bose-Einstein condensates consists of using a magnetic trap to collect and cool atoms [5]. To carry out this step, a ferromagnetic electromagnet must first be introduced, thereby creating very strong magnetic field gradients at the center of the structure (over the segment 0-α, see Fig. 3).

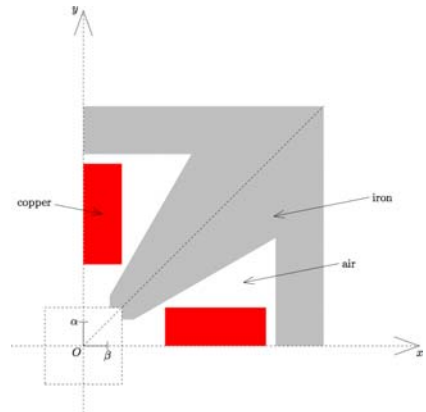


Fig. 3. Diagram of the electromagnet (1/4 of the structure shown)

For reasons of symmetry, we were able to proceed by meshing, using triangles, just one-eighth of the total electromagnet (Fig. 4). Each mesh element can contain air, iron or copper. Accordingly, our procedure calls for adopting integer values of 0, 1 and 2.

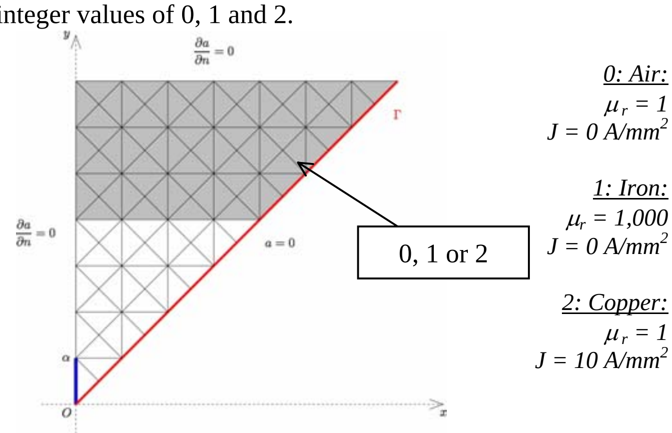


Fig. 4. Study domain

Determining the magnetic field associated with a given material distribution required computing potential vector $\vec{a}$ using the finite element code that accesses the *Melina*® library [6]:

$$rot\left(\frac{1}{\mu}\, rot\ a\right) = j \quad \text{with } b = rot\ a \qquad (1)$$

Boundary conditions have been indicated on Figure 4.

In the lower part of the mesh, as represented in white on Fig. 4, we now impose the presence of air to enable generating Bose-Einstein condensates.

The objective function to be maximized is the norm L^2 of the magnetic field along the blue edge $[O,\alpha]$ of Figure 4, given by:

$$\Delta = \sqrt{\int_{O\alpha} \left|\frac{\partial B_x}{\partial y}\right|^2 dy} \tag{2}$$

As illustrated in Figure 5, based on an imposed volume, the OMD method consists of determining the physical characteristic of each mesh element (i.e. a value of either 0, 1 or 2) in order to maximize Δ. The optimization routine proceeds by use of a genetic algorithm (NSGA II, [1]), whose primary settings are the constant values of mutation probabilities (pM) and crossover probabilities (pX). The influence of these parameters will be addressed in Section B below.

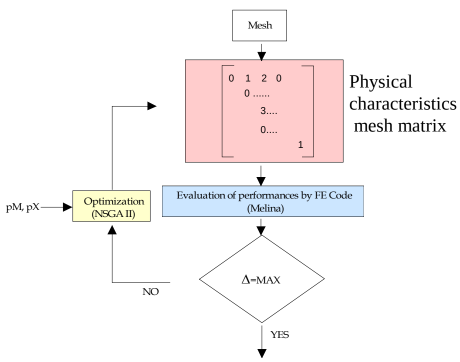


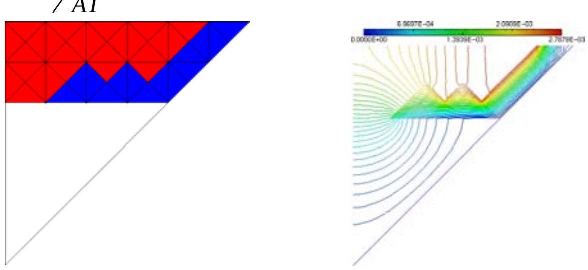
Fig. 5. Study domain

A. Reference case

To assess algorithm sensitivity to crossover probabilities pX and mutation probabilities pM, we considered a reference case comprising only a few variables: the mesh comprises just two lines above the white part, i.e. a total of 40 variables.

Moreover, the presumed values of pX and pM are both equal to 0.1. Each generation or iteration is composed of a population of 50 individuals.

Figure 6 depicts the optimal solution obtained for this reference case (the white part corresponds to air, the blue part to iron and the red part to copper). The distribution of induction B_x over segment 0-α is shown in Figure 7. The value of the corresponding objective function is: Δ=0.589; total ampere-turns equal $AT = \int_{\Omega} J.ds = 796.25$ A, which yields a ratio $\Delta/AT = 7.397 \times 10^{-4}$.



(a) (b)

Fig. 6. Optimal distribution of material (a) and flux density lines for pX=pM=0.1

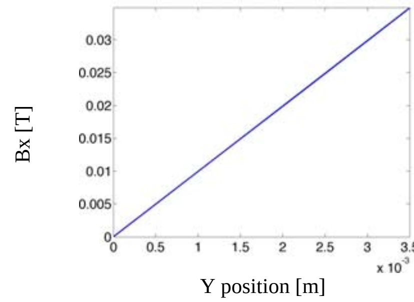
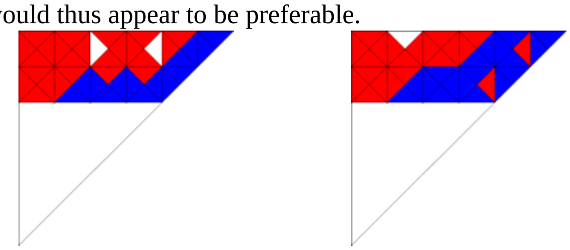


Fig. 7. Magnetic induction along $[O,\alpha]$ for pX=pM=0.1

B. Influence of mutation probability (pM)

Using the reference case as a starting point, we then modified the mutation probability (pM). The results derived for various pM values are displayed in Figures 8 and 9. According to expectations, as pM rises, convergence becomes increasingly difficult. After 200 generations, the optimal solution has still not been attained.

Choosing a very low mutation probability (pM < 0.1) would thus appear to be preferable.



(a) (b)

(c) (d)

Fig. 8. Optimal material distributions after 200 generations for pX=0.1 and (a) pM=0.3, (b) pM=0.5, (c) pM=0.7 and (d) pM=0.9

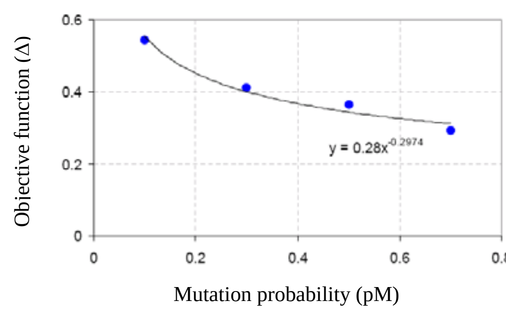


Fig. 9. Influence of mutation probability (pM) on the objective function for 100 generations and pX=0.1

C. Influence of crossover probabilities

This same study was also conducted to assess the sensitivity to crossover probability pX. The results, given in Figures 11 and 12, show that the genetic algorithm is relatively insensitive to pX. In the various cases observed on Figure 11 (pX=0.3, 0.5, 0.7 and 0.9), the optimal solution obtained as of the 100th generation lies very close to that of the reference case (pX=0.1).

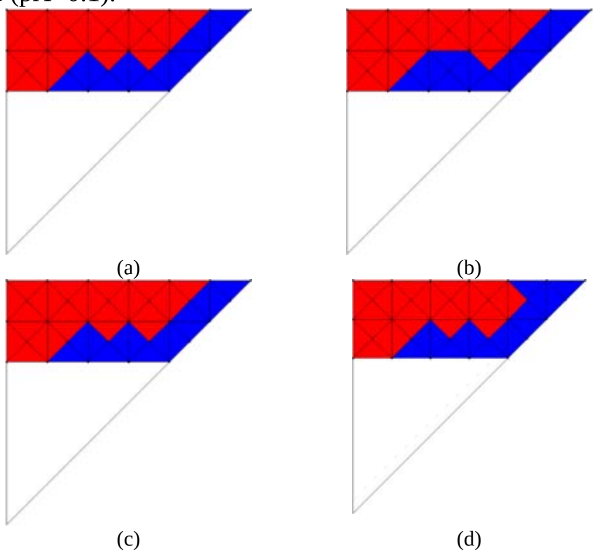


Fig. 10. Optimal material solution after 100 generations for pM=0.1 and (a) pX=0.3, (b) pX=0.5, (c) pX=0.7 and (d) pX=0.9

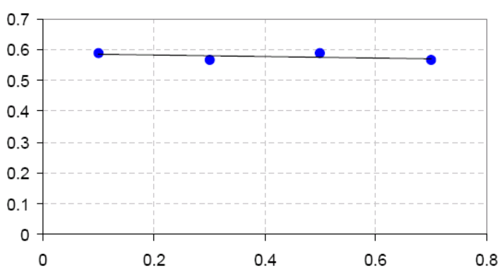


Fig. 11. Influence of crossover probability (pX) on the objective function for 100 generations and pM=0.1

D. *Number of variables*

The number of variables, correlated with the level of mesh refinement and hence with how finely the material has been distributed, exerts tremendous influence on the number of generations (or iterations) necessary and consequently on computation time. This consideration constitutes one of the chief problems inherent in the OMD method. For pX=pM=0.1, we performed a number of different optimizations by means of OMD, including an increasing number of variables (see Fig. 13). 25 generations were thus required to obtain an optimal solution with 18 variables, 100 generations with 40 variables, 400 with 66 variables and just over 1,000 with 96 variables (Fig. 14). In order to compress this computation time, particularly within an OMD strategy, it proved necessary to proceed step by step. During each incremental step, which contains a limited number of variables, an optimization sequence gets performed. The result obtained is then input as the initial solution into the subsequent step, which features a greater number of variables, and so forth.

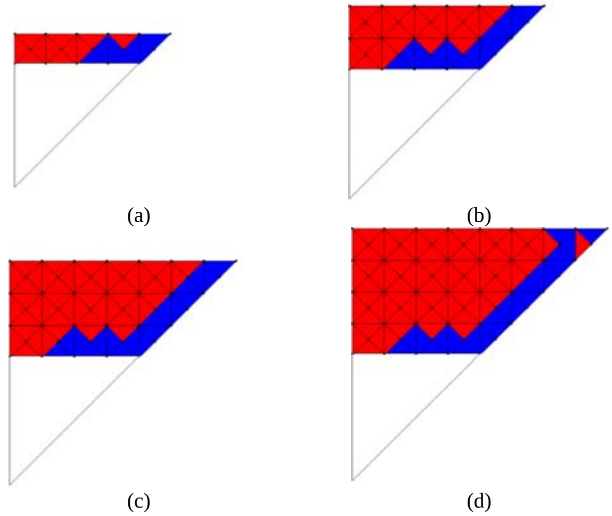


Fig. 12. Optimal material solution after: (a) 25 generations, (b) 100 generations, (c) 400 generations and (d) 1,000 generations

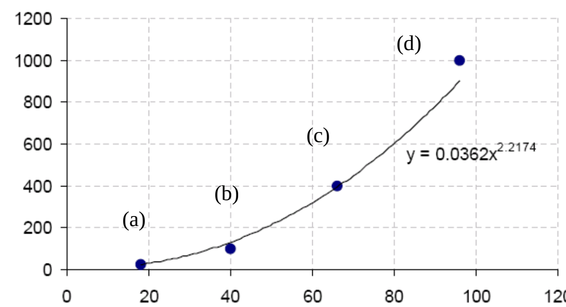


Fig. 13. Influence of the number of variables on the number of generations for pM=pX=0.1

E. *Comparison with a "classical" solution*

The optimized solution obtained using the OMD method has been compared with a "classical" solution derived by means of shape optimization. The optimal geometries for the two cases are indicated in Figure 14, and the corresponding flux density lines in figure 15.

The OMD method yielded better results, as regards both the objective function Δ and ratio Δ/AT (Fig. 16). The improvements realized with the OMD method are 3.5 and 2.3, respectively.

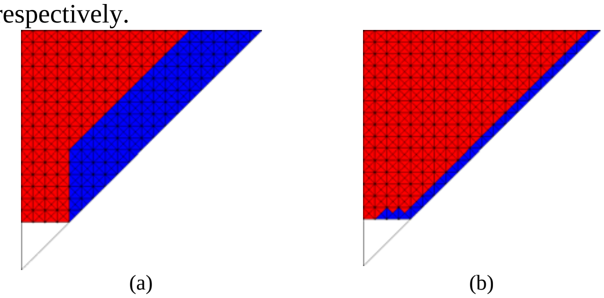
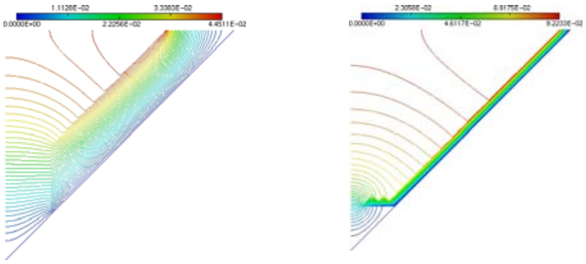


Fig. 14. Optimal classical configuration (a) and OMD configuration (b)



(a) (b)

Fig. 15. Corresponding flux density lines

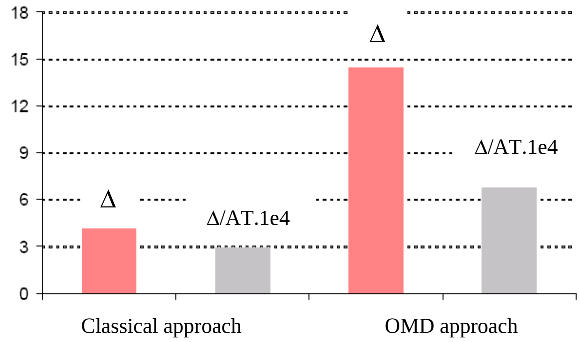


Fig. 16. Comparison of the two methods: classical and OMD

III. Other Examples

We will now succinctly describe other simple electromagnetic examples in which the OMD method has also been applied.

A. *Variable Reluctance Basic Actuator*

This first simple example (see Fig. 17) entails determining the optimal distribution of iron in an effort to maximize the quantity Δ, as given in (3), corresponding to the magnetic flux difference in both the direct and quadrature axes, for an imposed MMF ε.

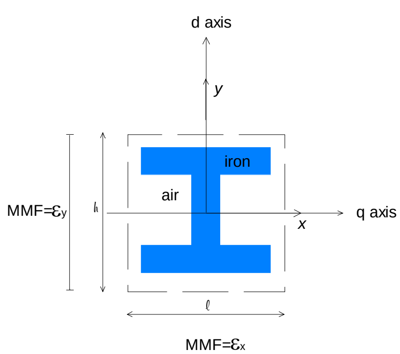
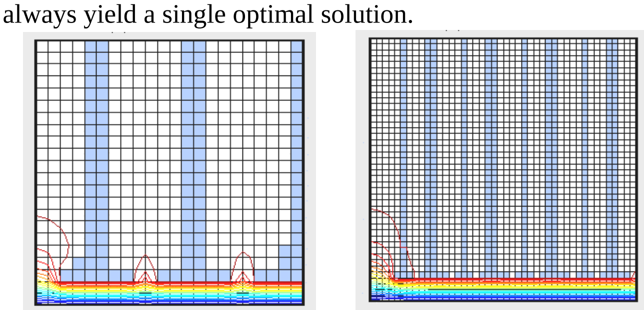


Fig. 17. Geometry of the VR Basic actuator

$$\Delta = \left| \begin{array}{l} \int_{-\ell/2}^{+\ell/2} B_y\left(x, \forall y, \varepsilon_x = 0, \varepsilon_y = \varepsilon\right).dx \\ -\int_{-h/2}^{+h/2} B_x\left(\forall x, y, \varepsilon_x = \varepsilon, \varepsilon_y = 0\right).dy \end{array} \right| \quad (3)$$

These results reveal that the "optimal" solution is intrinsically dependent upon the level of mesh refinement. The optimal iron distribution actually corresponds to the vertical rods whose thickness is proportional to the mesh link dimension. As the number of links increases, the number of rods also rises, as illustrated in Figure 18, in which the iron has been designated in blue and air in white.

This initial simple example has demonstrated that without any additional constraints, particularly on the technical feasibility of producing such a structure, OMD does not always yield a single optimal solution.



(a) (b)

Fig. 18. Optimal distribution: (a) coarse mesh, (b) fine mesh

B. *Variable Reluctance Repulsive Actuator*

In this second example, the focus lies in optimizing an electromagnet whose generated magnetic force, calculated from the Maxwell tensor, must be maximized and is known to be of the repulsion type (i.e. F < 0).

$$\Delta = -\int_S \frac{\mu_0}{2}\left[H_n^2 - H_t^2\right].ds \quad (4)$$

The initial geometry shown in Figure 19 serves to develop an attraction type force (F > 0), as is the case with most static electromagnets using solely iron and copper (no induced currents). This example then allows testing the OMD approach in terms of researching innovative concepts.

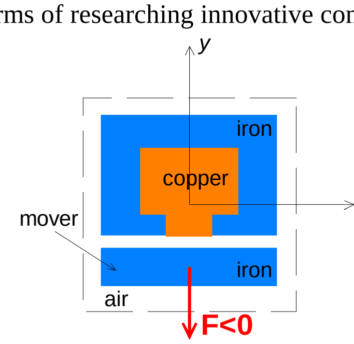


Fig. 19. Geometry of the VR repulsive actuator

Due to symmetry of the structure, just half of the geometry has been studied; results are displayed in Figure 20. Material optimization converges towards a solution for which the distribution of field lines within the airgap serves to generate a repulsive force, resulting from a sizable tangential component of the magnetic field (H_t) in comparison with the normal component (H_n). This procedure, as indicated in relation (4), yields a negative force (hence of the repulsive type) exerted on the mover.

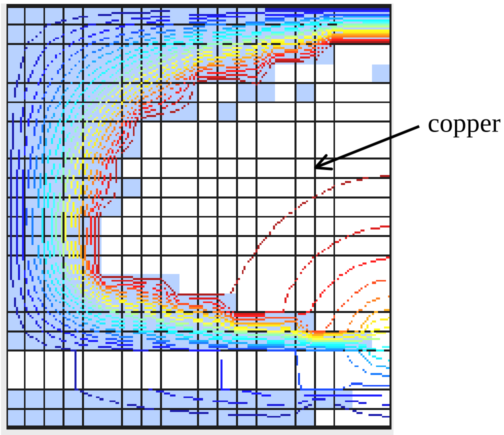


Fig. 20. Optimal distribution (for the VR repulsive actuator)

C. Variable Reluctance Linear Actuator

The third example corresponds to an elementary variable reluctance, i.e. a linear actuator. Iron and air compose the mobile part. The fixed part is made of iron and copper with a fixed current density (see Fig. 21). The objective function consists of maximizing the magnetic flux ratio between the conjunction and opposition positions (see Equation 5).

$$\Delta = \left|\varphi_{conj} - \varphi_{opp}\right| \quad (5)$$

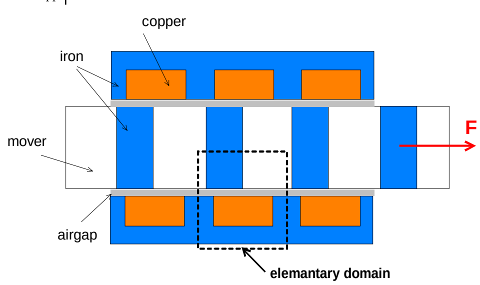


Fig. 21. Geometry of the VR linear actuator

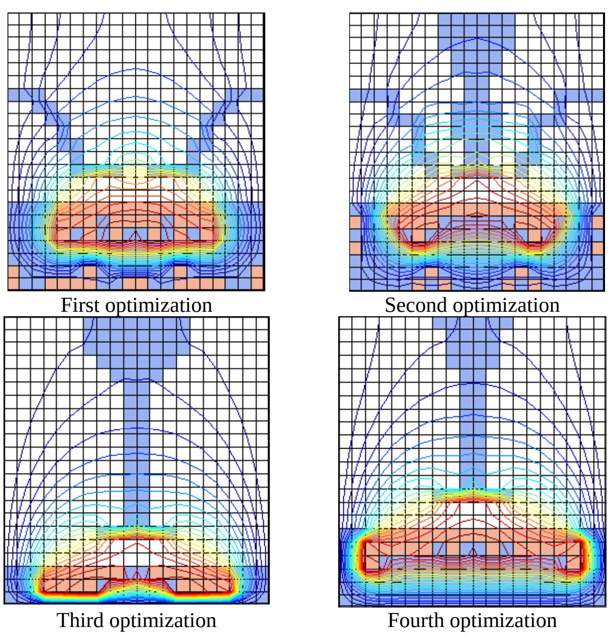


Fig. 22. Results from the preliminary optimization, in the opposition position

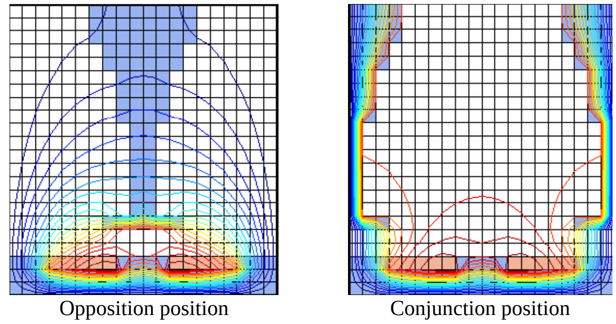


Fig. 23. Results from the final optimization

Several preliminary optimization steps have been performed in order to define the initial population of the final optimization. Preliminary optimization results, along with the final result, are presented in Figures 22 and 23 (the iron is denoted in blue, copper in red and air in white).

This final geometry enhances the benefit of splitting the copper coil to increase flux variation.

IV. Conclusion

The OMD method is based on overlapping the design and optimization processes. It does not entail searching for optimal geometric parameters or an optimized shape (along the lines of the classical approach), but instead seeks the optimal distribution of material by means of optimizing the physical characteristics of each mesh element. Applied to electromagnetic systems, it offers some outstanding and very attractive possibilities, yet carries the major disadvantage regarding the computation time required to reach a satisfactory solution, due to the very large number of variables the method needs to process. Consequently, OMD should be confined to solving simple or simplified problems that yield an initial optimal shape design that can then be refined using a more classical method.

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